IROS 2013

# Hybrid Force/Motion Control and Internal Dynamics of Quadrotors for Tool Operation

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Supported in part by the National Research Foundation (NRF), Korea
(2012-R1A2A2A0-1015797)

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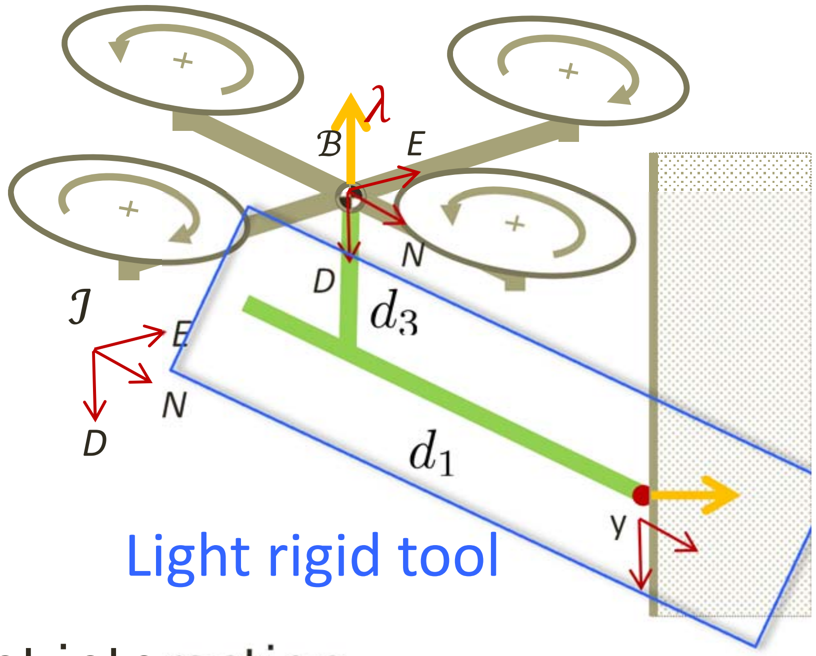
# Content

- Motivations
- Literature review
- Problem formulation
- Internal dynamics
- Simulation
- Conclusions and future work

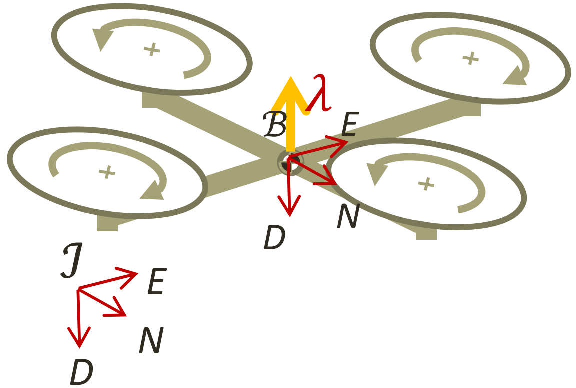
# Motivations




- Quadrotor with tool operation
  - Certain tasks require UAV-environment interaction
  - Payload limitation of UAV: prefer light rigid tool to multi-DOF arm

- Objective
  - Control the tool-tip y performing tasks (force/motion control)
  - Quadrotor as a versatile robotic platform

# Quadrotor control




*Motion control*

- Trajectory tracking control as M. *Hua 2009*
- Cooperative control as *N. Michael* 2011
- Distributed control as *D. Lee* 2012
- …

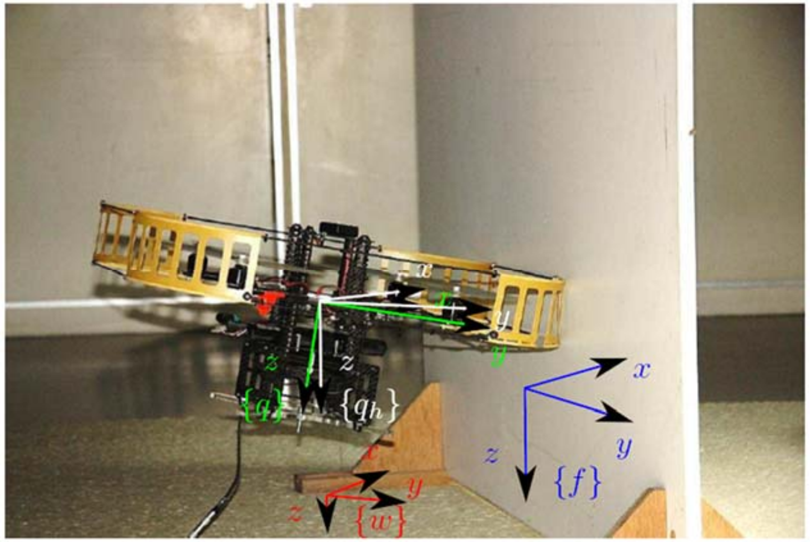
*Only few works on UAV-environment interaction*

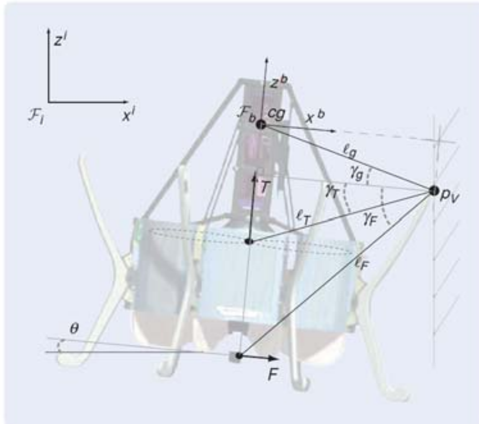
M.-D. Hua, T. Hamel, P. Morin, and C. Samson, "A Control Approach for Thrust-Propelled Underactuated Vehicles and its Application to VTOL Drones," *IEEE Transactions on Automatic Control,* vol. 54, pp. 1837-1853, 2009.

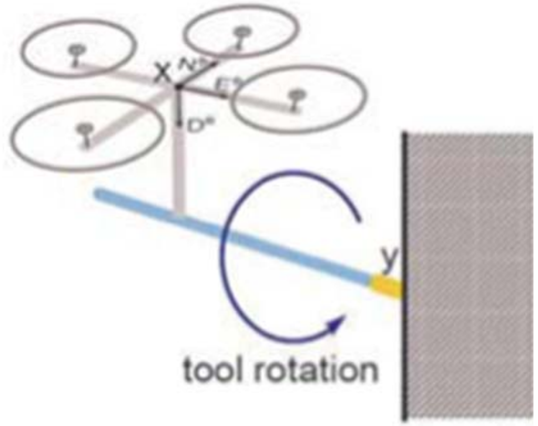
N. Michael, J. Fink, and V. Kumar, "Cooperative manipulation and transportation with aerial robots," *Autonomous Robots,* vol. 30, pp. 73-86, 2011.

D. Lee, "Distributed backstepping control of multiple thrust-propelled vehicles on a balanced graph," *Automatica,* vol. 48, pp. 2971-2977, 2012.

# UAV tool operation








Quasi-static wrench generator

Dynamics effect was not taken into account

S. Bellens, J. De Schutter, and H. Bruyninckx, "A hybrid pose / wrench control framework for quadrotor helicopters," in IEEE International Conference on Robotics and Automation (ICRA), 2012, pp. 2269-2274.

Planar Dynamics

L. Marconi and R. Naldi, "Control of Aerial Robots: Hybrid Force and Position Feedback for a Ducted Fan," *IEEE Control Systems Magazine,* vol. 32, pp. 43-65, 2012.

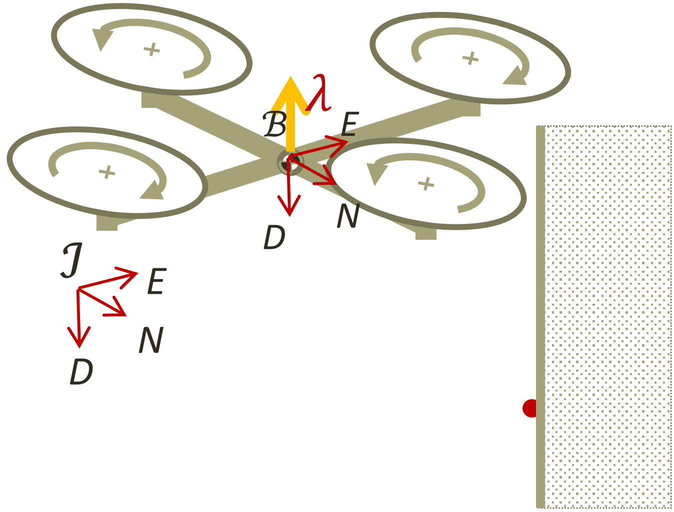
D. Lee and C. Ha, "Mechanics and control of quadrotor for tool operation," in ASME Dynamic Systems and Control Conference, 2012, pp. 177-184.

# Problem formulation

*Model*

$$m\ddot{x} = -\lambda R e_3 + mge_3 + f_e$$

$$J\dot{\omega} + \omega \times J\omega = \tau + \tau_e, \dot{R} = RS(\omega)$$





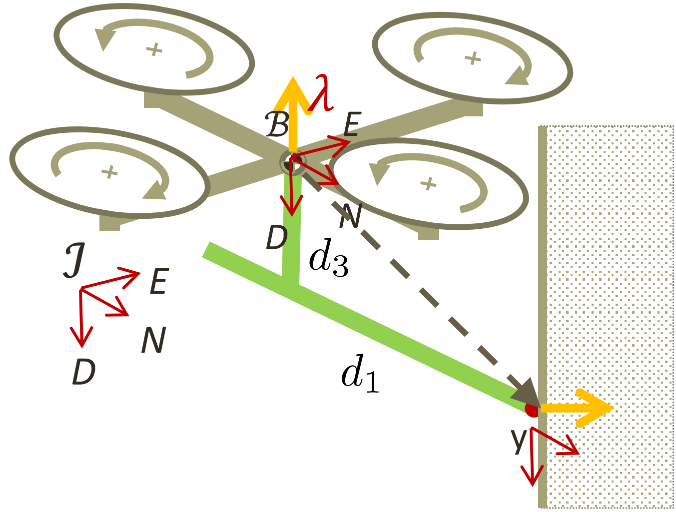
M.-D. Hua, T. Hamel, P. Morin, and C. Samson, "A Control Approach for Thrust-Propelled Underactuated Vehicles and its Application to VTOL Drones," *IEEE Transactions on Automatic Control,* vol. 54, pp. 1837-1853, 2009.

# Problem formulation

**Model** center-of-mass

$$m\ddot{x} = -\lambda R e_3 + mge_3 + f_e$$

$$J\dot{\omega} + \omega \times J\omega = \tau + \tau_e, \dot{R} = RS(\omega)$$




**Tool-tip position**

$$y = x + Rd$$

*y: tool-tip position in fixed frame $\mathcal{I}$*

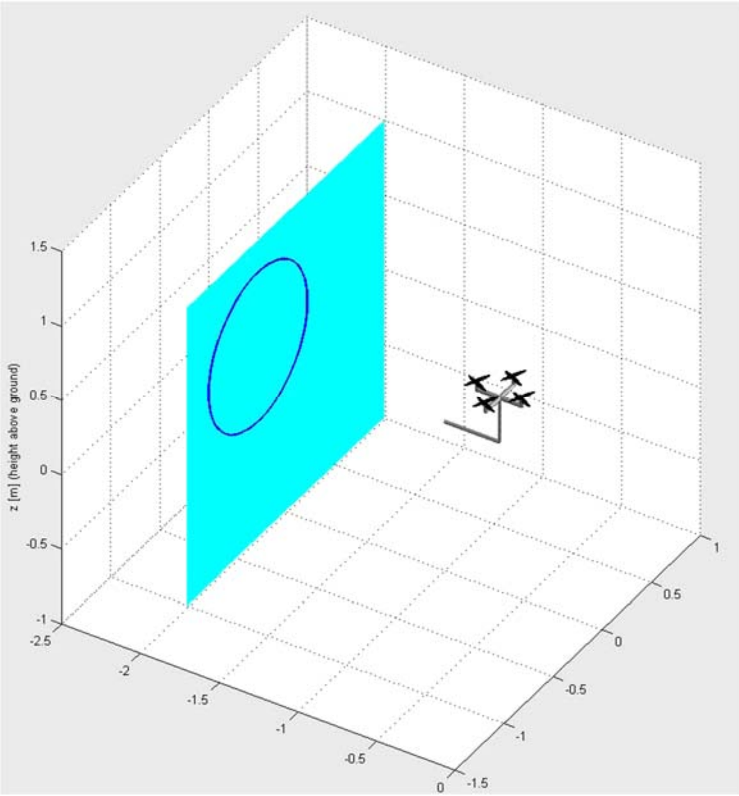
*d: tool-tip position in body frame $\mathcal{B}$*

*Objective – Hybrid force/motion control*

**Challenges**

Underactuated, yet, need to control center-of-mass position and rotation simultaneous to generate desired motion/force of y

# Contributions



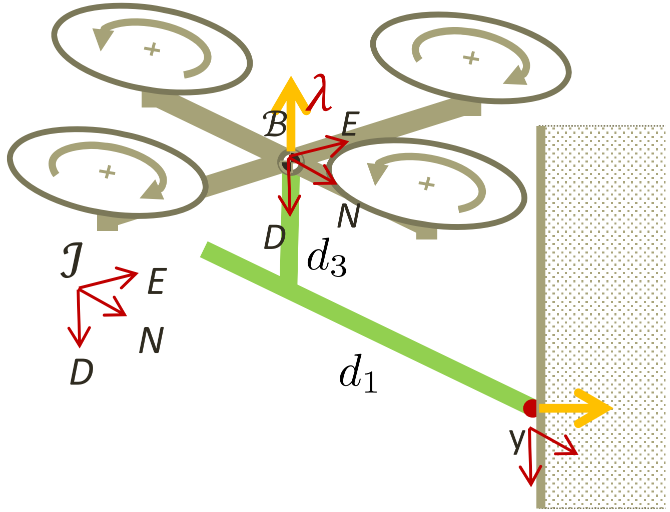
Hybrid force/motion control of quadrotor for tool operation

- Contributions
  - Reveal internal dynamics and elucidate structural condition necessary to internal dynamics stability
  - Propose controller to prevent finite-time escape
  - Develop hybrid force/motion control

Generalize the result of [Lee&Ha, DSCC12] for planar SE(2) motion to general SE(3) motion

D. Lee and C. Ha, "Mechanics and control of quadrotor for tool operation," in ASME Dynamic Systems and Control Conference, 2012, pp. 177-184.

# Problem formulation




## Model

center-of-mass; thrust control (1DoF)

$$m\ddot{x} = -\lambda R e_3 + m g e_3 + f_e$$

$$J\dot{\omega} + \omega \times J\omega = \tau + \tau_c, \; \dot{R} = RS(\omega)$$

control torque

## Tool-tip position

$$y = x + Rd$$

$$\ddot{x} = \ddot{y} - R\left[S(\dot{w}) + S^2(w)\right]d$$

*y: tool-tip position in fixed frame $\mathcal{I}$*

*d: tool-tip position in body frame $\mathcal{B}$*

*y-control needs simultaneous control of x and rotation R*

## y-dynamics

thrust control (1DoF)

$$m\ddot{y} = mR\left[S(\dot{w}) + S^2(w)\right]d - \lambda R e_3 + m g e_3 + f_e$$

$$J\dot{w} + w \times Jw = \tau + \tau_c, \quad \dot{R} = RS(w)$$

control torque

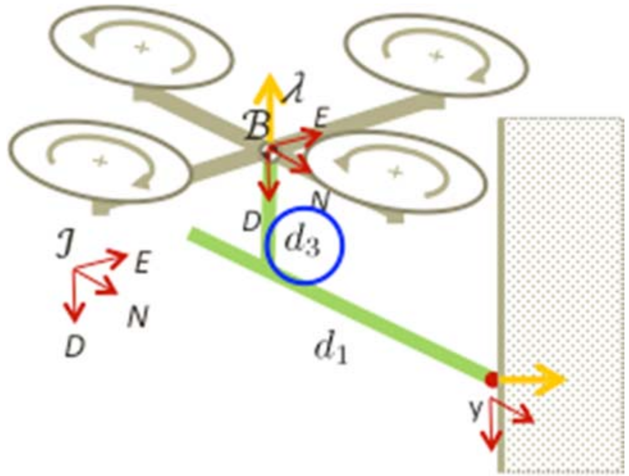
# Problem formulation

*y-dynamics*

Control inputs

$$m\ddot{y} = \underbrace{mR\left[S(\dot{w}) + S^2(w)\right]d - \lambda Re_3 + mge_3}_{:= u \in \Re^3} + f_e$$

$:= u \in \Re^3$ desired control




**Prop. 1** [DSCC12] Any control command *u* can be generated as long as $d_3 \neq 0$
(e.g., if d3=0, quadrotor cannot drive y-point forward instantaneously)

*Internal dynamics* Define dynamic relationship btw $\dot{\omega}, \omega$ and *R*

$$S(d)\dot{\omega} + S(\omega)S(d)\omega + \frac{\lambda}{m}e_3 - gR^T e_3 = -\hat{u}_d \qquad \hat{u}_d = \frac{1}{m}R^T u$$

Relation in so(3)

Relation in SO(3)

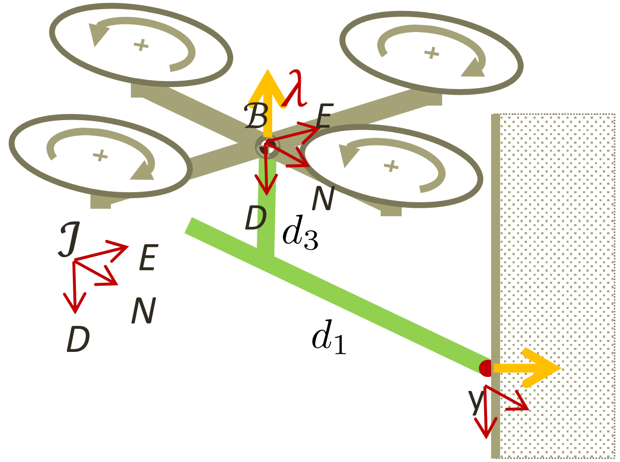
D. Lee and C. Ha, "Mechanics and control of quadrotor for tool operation," in ASME Dynamic Systems and Control Conference, 2012, pp. 177-184.

# Internal dynamics

u given

$$S(d)\dot{\omega} + S(\omega)S(d)\omega + \frac{\lambda}{m}e_3 - gR^T e_3 = -\hat{u}_d$$

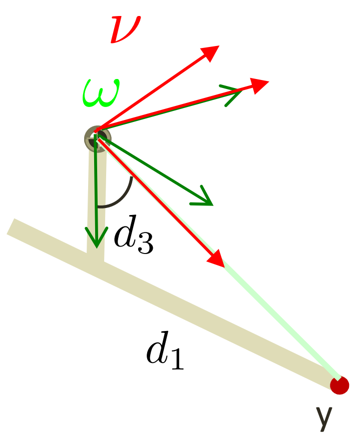
S(d) is singular quadratic term SO(3)




$$S(d) = \begin{bmatrix} 0 & -d_3 & 0 \\ d_3 & 0 & -d_1 \\ 0 & d_1 & 0 \end{bmatrix}$$

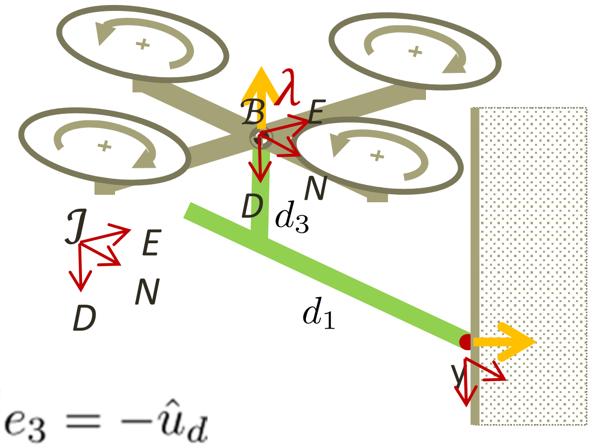
Using a coordinate transformation

$$\omega = [\Sigma_\top \quad \Sigma_\perp]\nu = \Sigma\nu \qquad \Sigma := \frac{1}{\alpha}\begin{bmatrix} -d_3 & 0 & d_1 \\ 0 & \alpha & 0 \\ d_1 & 0 & d_3 \end{bmatrix}$$




$$\begin{bmatrix} -d_3\dot{\nu}_2 \\ -\alpha\dot{\nu}_1 \\ d_1\dot{\nu}_2 \end{bmatrix} + \begin{bmatrix} d_1\nu_1^2 + d_1\nu_2^2 + d_3\nu_1\nu_3 \\ -\alpha\nu_2\nu_3 \\ d_3\nu_1^2 + d_3\nu_2^2 - d_1\nu_1\nu_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{\lambda}{m} \end{bmatrix} - gR^T e_3 = -\hat{u}_d$$

# Internal dynamics




## *Internal dynamics in $\nu$*

$$\begin{bmatrix} -d_3\dot{\nu}_2 \\ -\alpha\dot{\nu}_1 \\ d_1\dot{\nu}_2 \end{bmatrix} + \begin{bmatrix} d_1\nu_1^2 + d_1\nu_2^2 + d_3\nu_1\nu_3 \\ -\alpha\nu_2\nu_3 \\ d_3\nu_1^2 + d_3\nu_2^2 - d_1\nu_1\nu_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{\lambda}{m} \end{bmatrix} - gR^Te_3 = -\hat{u}_d$$

SO(3)

## *Local parameterization*

$$\frac{d}{dt}[\phi, \theta, \psi]^T = \Gamma\omega = \Gamma\Sigma\nu \qquad \Gamma(\theta, \psi) := \begin{bmatrix} 0 & \frac{\sin\psi}{\cos\theta} & \frac{\cos\psi}{\cos\theta} \\ 0 & \cos\psi & -\sin\psi \\ 1 & \frac{\sin\theta\sin\psi}{\cos\theta} & \frac{\sin\theta\cos\psi}{\cos\theta} \end{bmatrix}$$

## *Full Internal Dynamics*

$$\frac{d}{dt}(\theta, \psi, \nu_1, \nu_2)^T = \mathcal{F}(\theta, \psi, \nu_1, \nu_2) + (0, 0, \frac{1}{\alpha}\hat{u}_{d2}, \frac{1}{d_3}\hat{u}_{d1})^T$$

$$\mathcal{F}_1 = -\tfrac{d_1}{\alpha}\nu_1 \,\mathrm{s}\,\psi + \nu_2 \,\mathrm{c}\,\psi - \tfrac{d_3}{\alpha}\nu_3 \,\mathrm{s}\,\psi$$

$$\mathcal{F}_2 = -\tfrac{d_3}{\alpha}\nu_1 + \tfrac{d_1}{\alpha}\nu_1 \,\mathrm{t}\,\theta \,\mathrm{c}\,\psi + \nu_2 \,\mathrm{t}\,\theta \,\mathrm{s}\,\psi + \tfrac{d_1}{\alpha}\nu_3 + \tfrac{d_3}{\alpha} \,\mathrm{t}\,\theta \,\mathrm{c}\,\psi \nu_1\nu_3$$

$$\mathcal{F}_3 = -\nu_2\nu_3 - \tfrac{g}{\alpha} \,\mathrm{c}\,\theta \,\mathrm{s}\,\psi$$

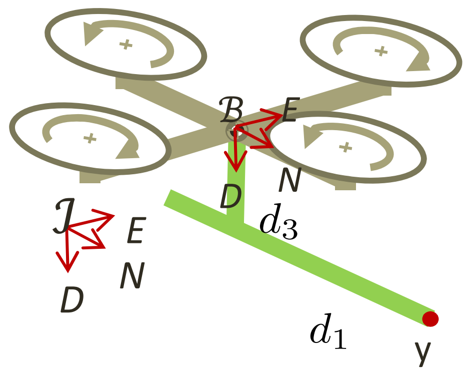
$$\mathcal{F}_4 = -\gamma(\nu_1^2 + \nu_2^2) + \nu_1\nu_3 + \tfrac{g}{d_3} \,\mathrm{s}\,\theta$$

# Linearized internal dynamics

$$\frac{d}{dt}(\theta, \psi, \nu_1, \nu_2)^T = \mathcal{F}(\theta, \psi, \nu_1, \nu_2) + (0, 0, \frac{1}{\alpha}\hat{u}_{d2}, \frac{1}{d_3}\hat{u}_{d1})^T$$

*Equilibrium and linearization*

$d_3 > 0$




At $[\theta, \psi, \nu_1, \nu_2] = [0, 0, 0, 0]$

$$\begin{bmatrix} \dot{\theta} \\ \dot{\psi} \\ \dot{\nu}_1 \\ \dot{\nu}_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -\frac{d_3}{\alpha} & 0 \\ 0 & -\frac{g}{\alpha} & 0 & 0 \\ \frac{g}{d_3} & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \theta \\ \psi \\ \nu_1 \\ \nu_2 \end{bmatrix}$$

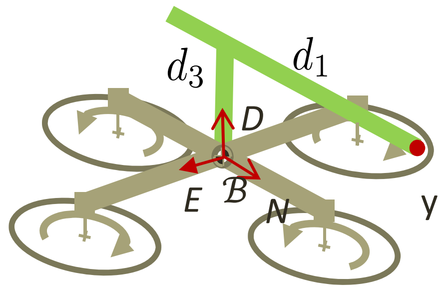
$$\lambda_1^2 = \frac{g}{d_3}, \quad \lambda_2^2 = \frac{gd_3}{\alpha^2}$$

unstable for positive $d_3$

At $[\theta, \psi, \nu_1, \nu_2] = [0, \pi, 0, 0]$

$$\lambda_1^2 = -\frac{g}{d_3}, \quad \lambda_2^2 = -\frac{gd_3}{\alpha^2}$$

unstable for negative $d_3$

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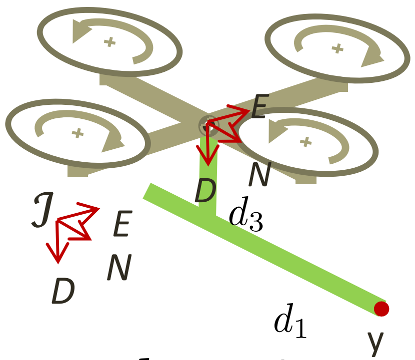
# Linearized internal dynamics

*Theorem 1:* A necessary condition for the internal dynamics stability at the equilibrium $(\theta, \psi, \nu_1, \nu_2) = 0$ is $d_3 < 0$ (i.e., tool attached above the quadrotor)

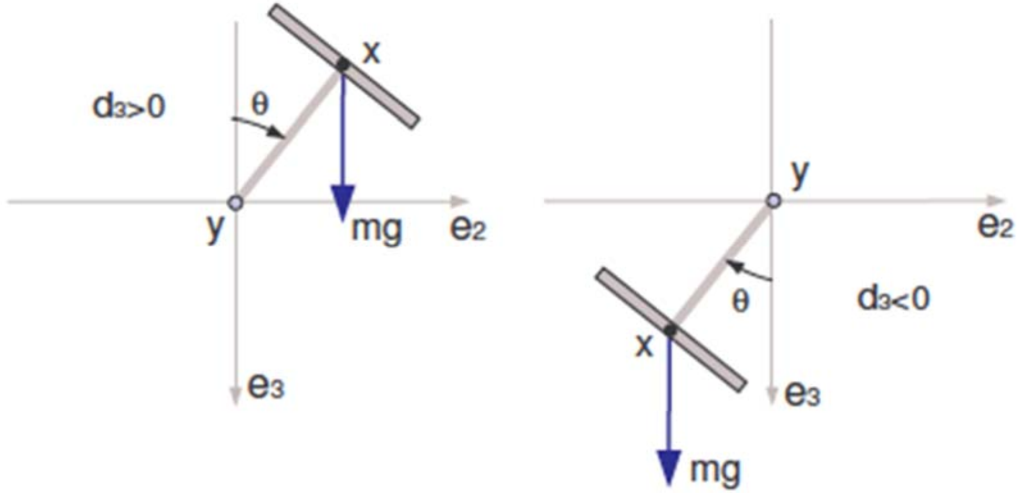



*Pendulum-like dynamic behavior*

At $[\theta, \psi, \nu_1, \nu_2] = [0, 0, 0, 0]$ and $\dot{\nu}_1 = 0$

$$\ddot{\theta} = -\gamma\dot{\theta}^2 + \frac{g}{d_3} sin\theta$$

$$\gamma = -\frac{d_1}{d_3}$$



# Finite-time escape

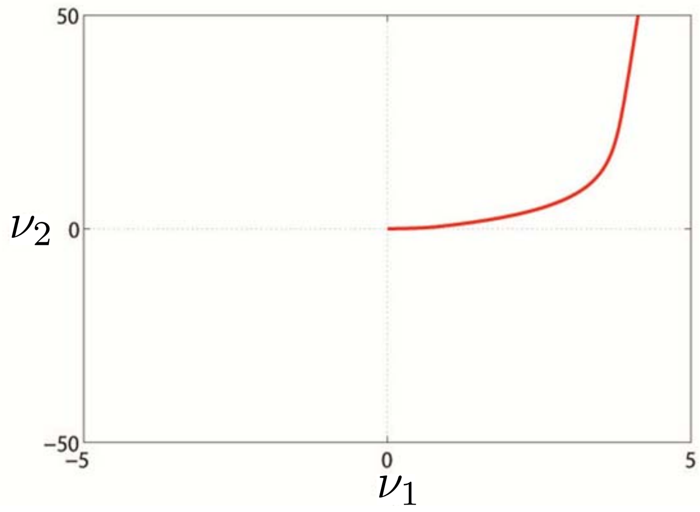
$$\begin{bmatrix} -d_3\dot{\nu}_2 \\ -\alpha\dot{\nu}_1 \\ d_1\dot{\nu}_2 \end{bmatrix} + \begin{bmatrix} d_1\nu_1^2 + d_1\nu_2^2 + d_3\nu_1\nu_3 \\ -\alpha\nu_2\nu_3 \\ d_3\nu_1^2 + d_3\nu_2^2 - d_1\nu_1\nu_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{\lambda}{m} \end{bmatrix} - gR^T e_3 = -\hat{u}_d$$

In angular velocity domain

$$\begin{bmatrix} \dot{\nu}_1 \\ \dot{\nu}_2 \end{bmatrix} = \begin{bmatrix} 0 \\ -\gamma(\nu_1^2 + \nu_2^2) \end{bmatrix} + \begin{bmatrix} -\nu_2 \\ \nu_1 \end{bmatrix} \nu_3 + \begin{pmatrix} \bar{u}_1 \\ \bar{u}_2 \end{pmatrix}$$

FINITE TIME ESCAPE

$$\gamma = -\frac{d_1}{d_3}$$



# Finite-time escape

$$\begin{bmatrix} -d_3\dot{\nu}_2 \\ -\alpha\dot{\nu}_1 \\ d_1\dot{\nu}_2 \end{bmatrix} + \begin{bmatrix} d_1\nu_1^2 + d_1\nu_2^2 + d_3\nu_1\nu_3 \\ -\alpha\nu_2\nu_3 \\ d_3\nu_1^2 + d_3\nu_2^2 - d_1\nu_1\nu_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{\lambda}{m} \end{bmatrix} - gR^T e_3 = -\hat{u}_d$$

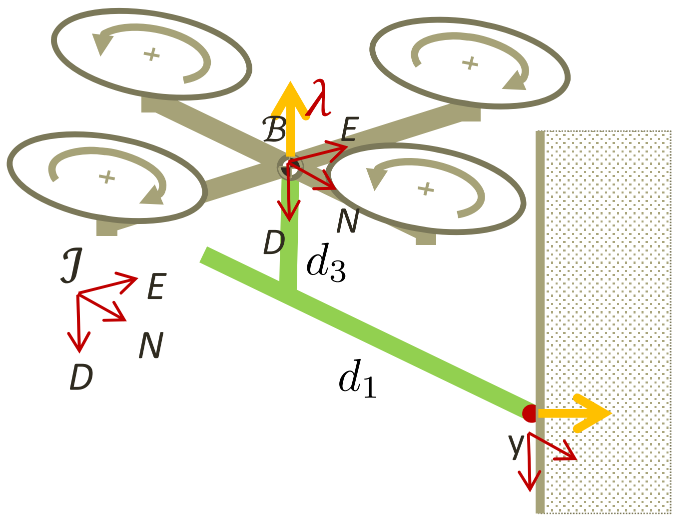
In angular velocity domain

$$\begin{bmatrix} \dot{\nu}_1 \\ \dot{\nu}_2 \end{bmatrix} = \begin{bmatrix} 0 \\ -\gamma(\nu_1^2 + \nu_2^2) \end{bmatrix} + \begin{bmatrix} -\nu_2 \\ \nu_1 \end{bmatrix} \nu_3 + \begin{pmatrix} \bar{u}_1 \\ \bar{u}_2 \end{pmatrix}$$

FINITE TIME ESCAPE

$$\gamma = -\frac{d_1}{d_3}$$

- Two mechanisms of internal instability:
  1) unstable if $d_3 > 0$ (tool below)
  2) finite-time escape
- Finite-escape more probable if tool is designed s.t., $d_1$ is longer than $d_3$



# Finite-time escape

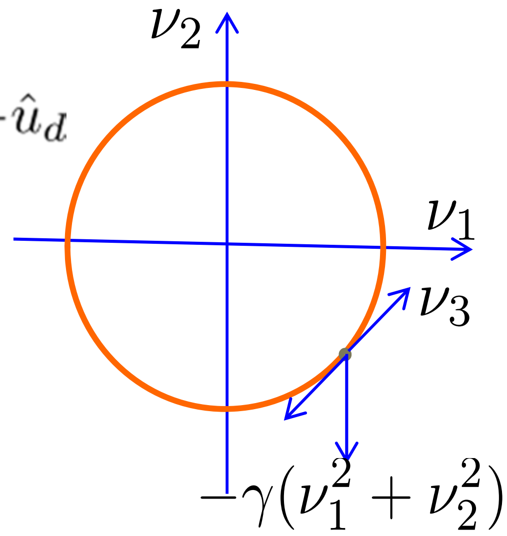
$$\begin{bmatrix} -d_3\dot{\nu}_2 \\ -\alpha\dot{\nu}_1 \\ d_1\dot{\nu}_2 \end{bmatrix} + \begin{bmatrix} d_1\nu_1^2 + d_1\nu_2^2 + d_3\nu_1\nu_3 \\ -\alpha\nu_2\nu_3 \\ d_3\nu_1^2 + d_3\nu_2^2 - d_1\nu_1\nu_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{\lambda}{m} \end{bmatrix} - gR^T e_3 = -\hat{u}_d$$

In angular velocity domain

$$\begin{bmatrix} \dot{\nu}_1 \\ \dot{\nu}_2 \end{bmatrix} = \begin{bmatrix} 0 \\ -\gamma(\nu_1^2 + \nu_2^2) \end{bmatrix} + \begin{bmatrix} -\nu_2 \\ \nu_1 \end{bmatrix} \nu_3 + \begin{pmatrix} \bar{u}_1 \\ \bar{u}_2 \end{pmatrix}$$

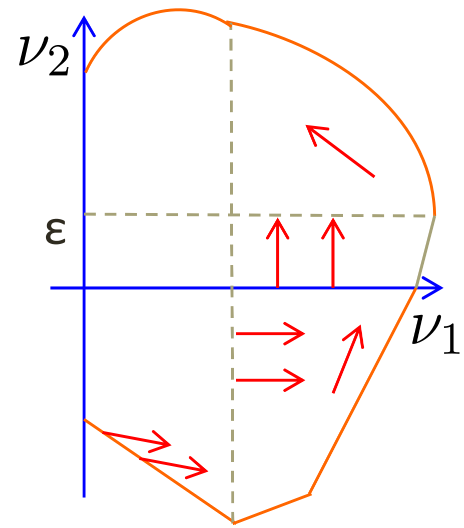
FINITE TIME ESCAPE

Only act on tangent direction

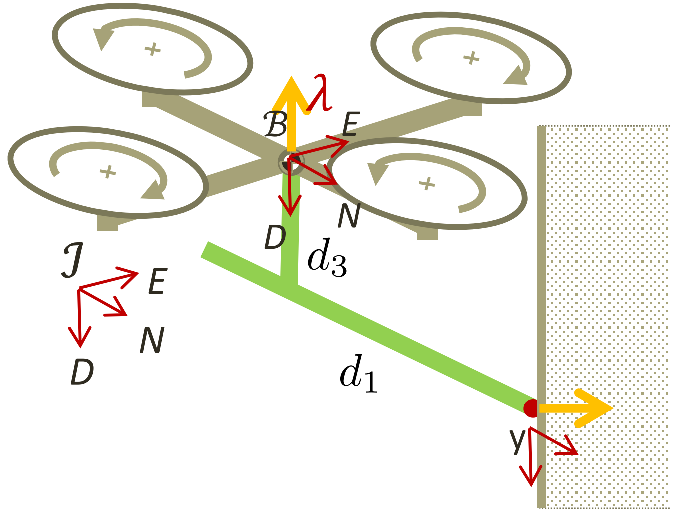



Propose: Finite-time escape preventing action (in the sense of boundedness of $\nu$)

$$\nu_3 = k\nu_1(1 + \nu_2^2) \qquad k \geq \gamma$$



# Hybrid force/motion control




**Model** $m\ddot{y} = u + f_e$

$$h(y) = 0 \quad \frac{\partial h}{\partial y}\dot{y} = 0$$

*Hybrid force/motion control require the contact to be maintained all the time, which is not practical*

**Passive decomposition** [Lee&Li, IEEE T-AC2013]

$$\dot{y} = \Delta v = [\Delta_\top \quad \Delta_\perp] \begin{pmatrix} v_l \\ v_h \end{pmatrix} \quad u + f_e = [\Omega_\top^T \quad \Omega_\perp^T] \begin{pmatrix} u_l + f_l \\ u_h + f_h \end{pmatrix}$$

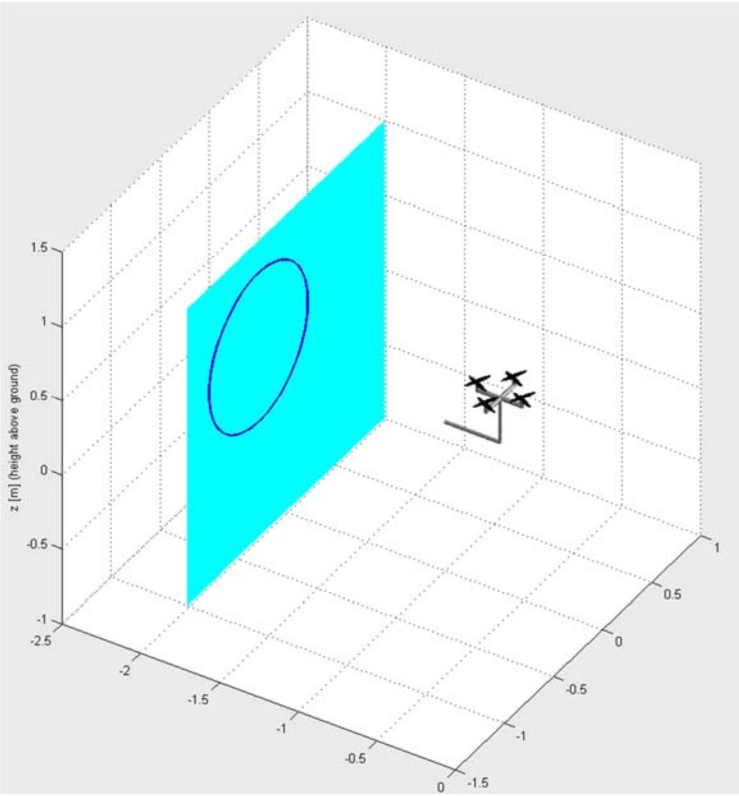
$$m\dot{v}_l + Q_l v_l + Q_{lh} v_h = u_l + f_l$$

$$m\dot{v}_h + Q_h v_h + Q_{hl} v_l = u_h + f_h$$

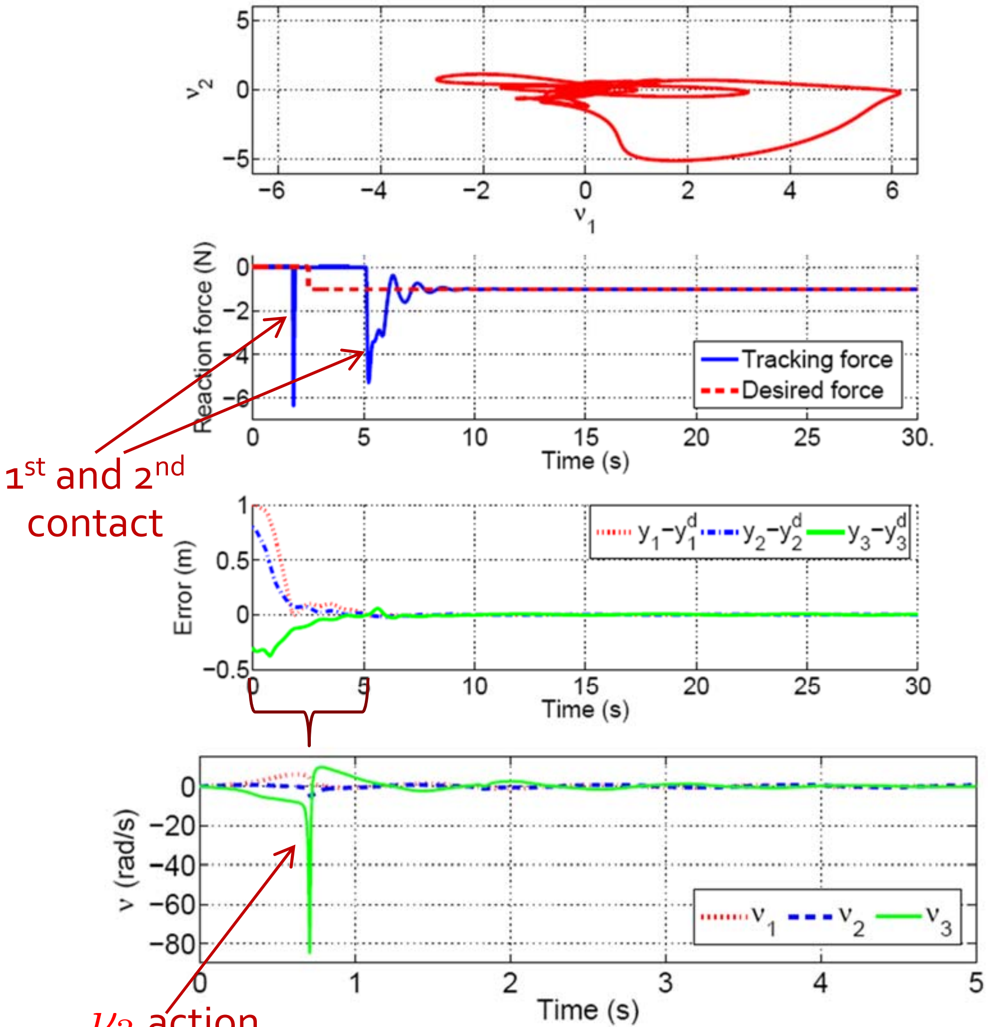
The system is decomposed into tangential motion subspace and normal force subspaces and then suitably controlled.

D. Lee and P. Y. Li, "Passive decomposition of multiple mechanical systems under motion coordination requirements," IEEE Transactions on Automatic Control, vol. 58, pp. 230–235, 2013.

# Simulation



Upside down effect
$d = [0.35; 0; 0.3]$

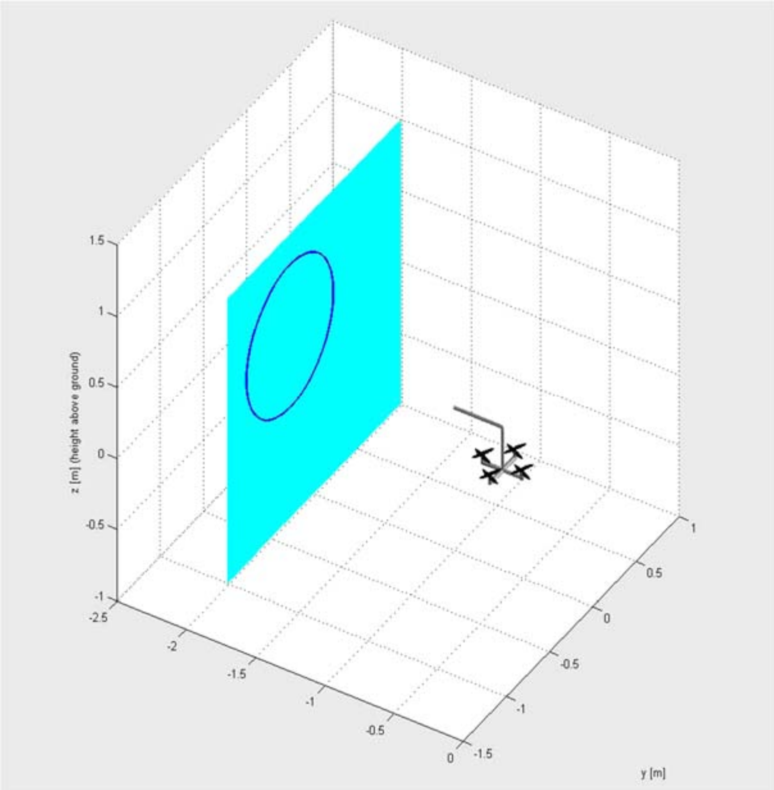


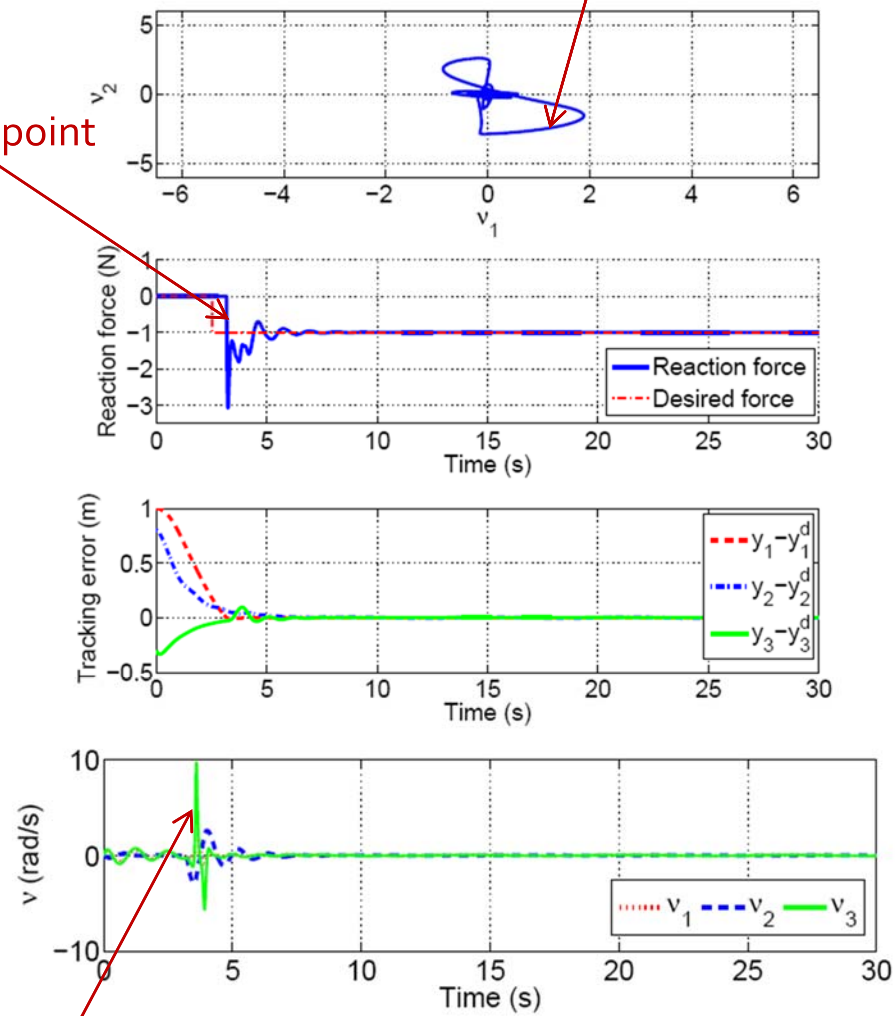

*Upside down effect*

# Simulation

prevent finite-time escape effect

Contact point



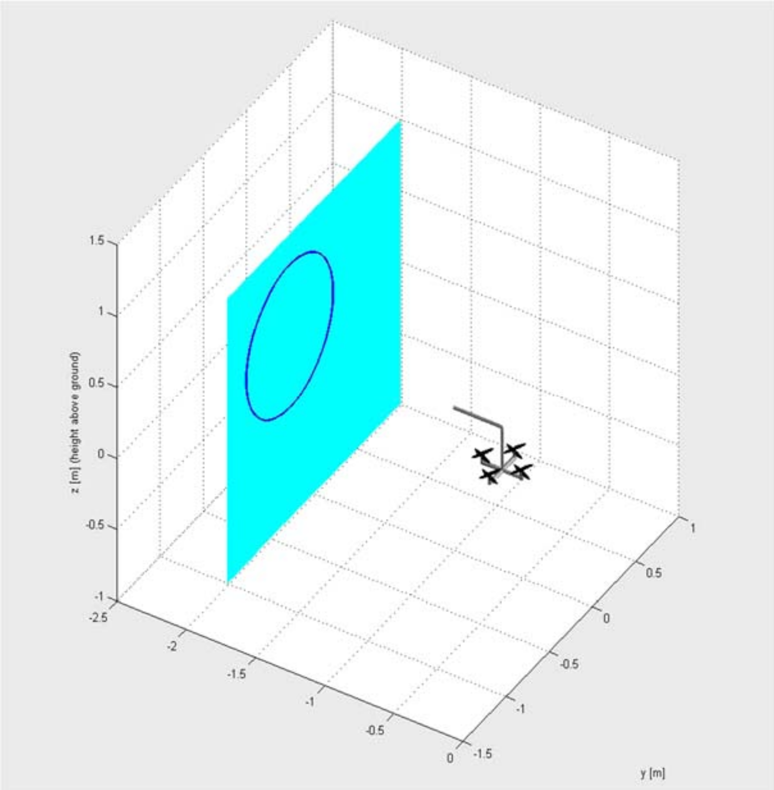




Hybrid force/motion control of quadrotor for tool operation

$d = [0.35; 0; -0.3]$:

$\nu_3$ action

# Conclusions and future work




*Physical interaction*

- Reveal internal dynamics with necessary condition
- Propose a control action to prevent finite-time escape
- Use passive decomposition for hybrid force/motion control

*Future work*

- Collision-avoidance
- Consider disturbance
- Experimental implementation

*Thank you*

*for your attention!*